

Q1.

A wheel of diameter 3.0 cm has a 4.0 m cord wrapped around its periphery starting from rest, the wheel is given a constant angular acceleration of 2.0 rad/s^2 . How much time it will take for the cord to unwind?

Ans.

$$\theta = \omega_i t + \frac{1}{2} \alpha t^2$$

$$\text{but } \omega_i = 0 \text{ so } \theta = \frac{1}{2} \alpha t^2 \text{ --- (1)}$$

$$\theta = \frac{S}{r}; S = 4 \text{ m and } r = \frac{3.0}{2} = 1.5 \text{ cm} = 0.015 \text{ m}$$

$$\theta = \frac{4}{0.015} = 267 \text{ rad}$$

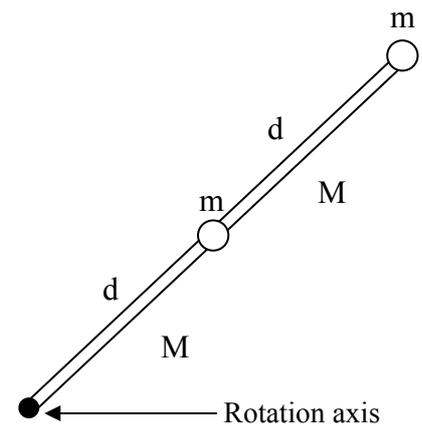
Using this value of θ in eq. (1)

$$267 = \frac{1}{2} \times 2.0 \times t^2 \Rightarrow t^2 = 267$$

$$t = \sqrt{267} = 16.35 \text{ s}$$

Q2.

As shown in the figure, two particles each of mass $m=1.5 \text{ kg}$, are fastened to each other, and to a rotation axis at O by two identical thin rods with length $d= 5.0 \text{ cm}$ and mass $M= 1.5 \text{ kg}$. The combination rotates around the rotation axis with angular speed $\omega=0.5 \text{ rad/s}$. Calculate their kinetic energy?

**Ans**

$$I = I_{rod1} + I_{mass-1} + I_{rod-2} + I_{mass-2}$$

$$I_{rod-1} = \frac{1}{12} (M)(d)^2 + M\left(\frac{d}{2}\right)^2 = \frac{1}{3} Md^2$$

$$I_{rod-2} = \frac{1}{12} (M)(d)^2 + M\left(\frac{3d}{2}\right)^2 = \frac{7}{3} Md^2$$

$$I_{mass-1} = md^2 = md^2$$

$$I_{mass-2} = m(2d)^2 = 4md^2$$

$$I = \frac{1}{3} Md^2 + md^2 + \frac{7}{3} Md^2 + 4md^2 = \frac{23}{3} md^2 \quad \text{since } M = m$$

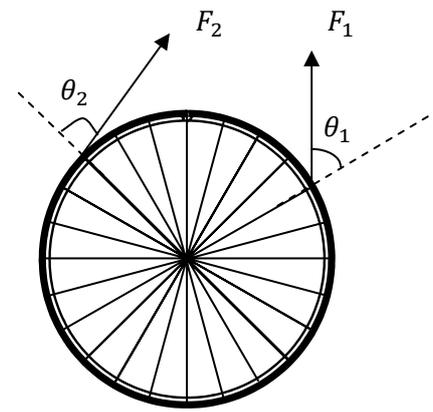
$$= \frac{23}{3} \times 1.5 \times (0.05)^2$$

$$I = 0.0288 \text{ kg m}^2$$

$$K = \frac{1}{2} I \omega^2 = 0.5 \times 0.0288 \times 0.5^2 = 3.6 \times 10^{-3} \text{ J}$$

Q3.

Two forces act on a wheel, as shown in the figure. The wheel is free to rotate about its fixed axis without friction, has a radius of 0.5 m, and is initially at rest. Given that $F_1=12\text{N}$, $F_2=10\text{N}$, $\theta_1=50^\circ$ and $\theta_2=70^\circ$ find the net torque caused by the two forces.

**Ans.**

$$\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2$$

τ_1 is the torque due to force F_1 and it rotates the wheel in counter clockwise directions whereas τ_2 is the torque by force F_2 and it rotates the wheel in clockwise direction. So τ_1 is positive and τ_2 is negative. And the net torque τ is given as:

$$\tau = RF_1 \sin\theta_1 - RF_2 \sin\theta_2$$

$$\tau = 0.5 \times 12 \sin 50 - 0.5 \times 10 \sin 70 = -0.1 \text{ N.m}$$