MATH 102 (Semester 102)

Review: Power Series

1. *Basic Definitions:*

   - A *Power Series* is an infinite series of the form

   $$\sum_{n=0}^{\infty} c_n(x-a)^n$$

   where c_n are the **coefficients** and a is the **centre** of the series. A power series may converge for some values of x and not others.

   - The **interval of convergence** is the set of points x that make the series converge. There are three possibilities for the interval of convergence. It may be
     (a) the entire real line (the series converges everywhere),
     (b) a finite interval (open or closed at either end) centred at the point a,
     (c) the single point $\{a\}$.
   - The **radius of convergence** R is the distance from the centre a to the end-points of the interval. It is ∞ if the interval of convergence is the entire line, and 0 if the interval of convergence is a single point.
   - The radius and interval of covergence are usually found using the **Ratio Test**. The convergence at the endpoints of the interval is determined by substituting the endpoints back into the original series.

2. *Functions as Power Series:*

   - The **Taylor Series** of a function $f(x)$ about a point a is given by the formula

   $$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

   If $a = 0$ the series is called the **MacLaurin Series**.

   \- **Examples** of Maclaurin series are:
     - $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ $\qquad x \in (-1, 1)$
     - $\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$ $\qquad x \in (-\infty, \infty)$
     - $\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ $\qquad x \in (-\infty, \infty)$
     - $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ $\qquad x \in (-\infty, \infty)$
     - $\tan^{-1}(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$ $\qquad x \in (-1, 1)$
     - $(1+x)^n = \sum_{n=0}^{\infty} \binom{k}{n} x^n$ $\qquad x \in (-1, 1)$

   where $\binom{k}{n} = \frac{k(k-1)(k-2)\ldots(k-n+1)}{n!}$.