Summary of Convergence Tests

NAME	STATEMENT	COMMENTS
Divergence Test	If $\lim_{k\to\infty} u_k \neq 0$,then $\sum u_k$ diverges.	If $\lim_{k\to\infty} u_k = 0$, then$\sum u_k$ mayor may not converge.
Integral Test	Let $\sum u_k$be a series with positive terms, and let $f(x)$ be the function that results when k is replaced by x in the general term of the series decreasing and continuous for $x \geq a$,then $$\sum_{k\to 1}^{\infty} u_k \ and \int_a^{+\infty} f(x)dx$$ both converge or both diverge.	This test only applies to series that have positive terms. Try this test when $f(x)$ is easy to integrate.
Comparison Test	Let $\sum_{k=1}^{\infty} a_k \ \ and \ \ \sum_{k=1}^{\infty} b_k$ be series with nonnegative terms such that $$a_1 \leq b_1, a_2 \leq b_2, \dots, a_k \leq b_k, \dots$$ If $\sum_{k=1}^{\infty} b_k$ converges, then $\sum_{k=1}^{\infty} a_k$ converges, and if $\sum_{k=1}^{\infty} a_k$ diverges, then$\sum_{k=1}^{\infty} b_k$ diverges.	This test only applies to series with nonnegative terms. Try this test as a last resort: other tests are often easier to apply.
Limit Comparison Test	Let $\sum a_k \ and \ \sum b_k$ be series with positive terms such that $p = \lim_{k\to+\infty} \frac{a_k}{b_k}$ If $o < p < +\infty$ then both series converge or diverge.	This easier to apply than the comparison test, but still requires some skills in choosing the series $\sum b_k$ for comparison.
Ratio Test	Let $\sum u_k$ be a series with positive terms and suppose $$p = \lim_{k\to+\infty} \frac{u_k + 1}{u_k}$$ (a) Series converge if $p < 1$ (b) Series diverges if $p > 1 \ or \ p = +\infty$ (c) The test is inconclusive if $p = 1$.	Try this test when u_k involves factorials or kth power.
Root Test	Let $\sum u_k$ be a series with positive terms such that $$p = \lim_{k\to\infty} \sqrt[k]{u_k}$$ (a) Series converge if $p < 1$ (b) Series diverges if $p > 1 \ or \ p = +\infty$ (c) The test is inconclusive if $p = 1$.	Try this test when u_k involves kth power.
Alternating Series Test	If $a_k > 0 \ for \ k = 1,2,3, \dots$, then the series $$a_1 - a_2 + a_3 - a_4 +$$ $$-a_1 + a_2 - a_3 + a_4 -$$ Converge if the following condition hold : (a) $a_1 \geq a_2 \geq a_3 \geq a_4$ (b) If $\lim_{k\to\infty} a_k = 0$.	This test applies only to alternating series.
Ratio Test for Absolute Convergence	Let $\sum u_k$ be a series with positive terms such that $$p = \lim_{k\to+\infty} \frac{\lvert u_k + 1\rvert}{\lvert u_k\rvert}$$ (a) Series converge if $p < 1$ (b) Series diverges if $p > 1 \ or \ p = +\infty$ (c) The test is inconclusive if $p = 1$.	The series need not have positive terms and need not be alternating to use this test.

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